Progressive Education Society's
Modern College of Arts, Science & Commerce Ganeshkhind, Pune – 16
End Semester Examination – January 2023
Faculty : Science and Technology

Program : B.Sc. Code (BScGen03) **Semester : I** **SET: C**
Program (Specific) : Mathematics **Course Type : Core Course**
Class : F.Y.B.Sc.(Regular) **Max.Marks : 35**
Name of the Course : Algebra
Course Code : 22-MT-111 **Time : 2Hr**
Paper : I

Instructions to the candidate:

1) ***There are 4 sections in the question paper. Write each section on separate page.***
2) ***All Sections are compulsory.***
3) ***Figures to the right indicate full marks.***
4) ***Draw a well labelled diagram wherever necessary.***

SECTION : A

Q1) Select correct alternative. **[Marks 5]**

i) A function $f : X \to Y$ is such that $f(x) = c$, where c is constant is called as

a) constant function b) inverse function c) inclusion function d) bijective function

ii) If $4x \equiv 2 (\text{mod } 6)$ then the value of x is

a) 3 b) 5 c) 6 d) 9

iii) The remainder of 10^{12} when divided by 13 is

a) 12 b) 13 c) 1 d) 0

iv) If f(x) and g(x) be the polynomials over $\mathbb{R}$ such that degree of f(x) is n and degree of g(x) is m then deg(f(x) + g(x)) =

a) n + m b) n - m c) nm d) Max { n , m}

v) If z_1 and z_2 be two complex numbers then $\arg(z_1 / z_2)$ =

a) $\arg z_1 \cdot \arg z_2$ b) $\arg(z_1 + z_2)$ c) $\arg z_1 + \arg z_2$ d) $\arg z_1 - \arg z_2$

Q2) Attempt any four. **[Marks 4]**

i) Define symmetric relation.

ii) If $X = \{ -5 , -3 , 0 , 2 , 4 \}$ and $f : X \to \mathbb{R}$ given by $f(x) = 2x + 3$, then find the range of the function f.

iii) $3x \equiv 3 \pmod 8$ then find the value of x

iv) In $\mathbb{Z}_{11}$, find the value of $\bar{7} \cdot (\bar{6} + \bar{5})$

v) State Factor Theorem.

vi) Find modulus of the complex number $z = 5 + 4i$

SECTION: B

Q3) Attempt any four. **[Marks 8]**

i) Define equivalence relation.

ii) If $a \mid b$ and $a \mid c$ then show that $a \mid bx + cy$

iii) Which elements of $\mathbb{Z}_8$ satisfy the equation $x^2 = x$

iv) If $f(x) = x^4 - 7x^3 + 2x^2 - 8$, $g(x) = x^2 + x + 3$ then find a) $f(x) + g(x)$, b) $f(x) \cdot g(x)$

v) Using the synthetic division find the quotient and remainder when $7x^4 + 3x^3 + 2x + 1$ is divided by $x + 4$

vi) Express $z = (1 - 3i)(1 - 5i)$ in the form $x + iy$. Find the values of x and y

SECTION: C

Q4) Attempt any four. **[Marks 8]**

i) Define composite function .

ii) If $a \equiv b \pmod n$ and $c \equiv d \pmod n$ then show that $(ax + cy) \equiv (bx + dy) \pmod n$

iii) Find all pairs $\bar{\imath}$ and $\bar{\jmath}$ in Z_8 such that $\bar{\imath} \cdot \bar{\jmath} = \bar{1}$

iv) Find all primes which divide 40!

v) If 2 is a root of the equation $x^3 + 4x^2 + ax + 4 = 0$ then find the value of a

vi) State De Moiver's theorem.

SECTION: D

Q5) Attempt any two. **[Marks 10]**

i) Find g.c.d. of 1234 and 111 and express it in the form $1234\,m + 111\,n$.
Find the value of m and n.

ii) Prepare the composition table with respect to addition for the set $\mathbb{Z}_6$

iii) If the two roots of the equation $4x^3 - 12x^2 - 15x - 4 = 0$ are equal then find all roots.

iv) Express $\cos 5\theta$ and $\sin 5\theta$ in terms of cos and sin